Pathway2Careers
Algebra II

Pathway2Careers Algebra II

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As NS4ed's flagship program, Pathway2Careers (P2C) is a career readiness solution that support schools in their efforts to prepare students for high-value careers in their communities. P2C delivers access to labor market data to encourage the use of data-informed practices in career education through a number of services, including labor market data tools, career-focused curricula, Perkins V support, and more. As a comprehensive career readiness solution, P2C serves administrators, faculty, students, and parents in making informed decisions that promote employment success.

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** "S" denotes that the lesson is a Supporting Lesson and may be omitted or abbreviated a the discression of the teacher/district. Pacing for these lessons is not included in the chapter or course totals.

** "E" denotes that the lesson is an Enrichment Lesson and may be omitted or abbreviated a the discression of the teacher/district. Pacing for these lessons is not included in the chapter or course totals.

LESSON 1.2

Isolating a Variable

CAREER PREPARATION: Essential Algebra II Skills

Did you know?

A police officer would isolate a variable in order to calculate your average rate of speed.

Consider this situation…

Suppose a car that is equipped with an E-Z Pass drives from the toll plaza in Carlisle, PA (milage marker 226 on the PA Turnpike) to the one in Valley Forge (marker 326) in 1 hour and 15 minutes.

What was the average speed of the car?

This problem will use the formula $d = rt$. Where *d* = distance travelled, *r* = average rate of speed and *t* = time. Since we are looking for the value of *r*, we can solve this equation for *r*.

$d = rt$ divide both sides by t

$\frac{d}{t} = \frac{rt}{t}$ on the right side of the equation, the *t* would cancel

$\frac{d}{t} = r$

If t = 0, you could not divide by t.

Now, we can discover the average speed of the car: distance = $326 - 226 = 100$ and time = $1\frac{15}{60} = 1.25$ hours. The car's average speed was $\frac{100}{1.25} = 80$ mph.

Lesson Objective

In this lesson, you will learn how to:

- Rearrange formulas to highlight a quantity of interest in multi-step equations.

Algebra II Essentials

When given a formula with multiple steps, it is helpful to rearrange the formula to solve for the variable of interest before plugging numbers into the equation.

- To solve an equation for a given variable, you must isolate the variable on one side of the equation.
- To isolate the variable, use the Properties of Equality from lesson 1 to rearrange the equation.

Solving a Formula for a Given Variable

Example 1 Solve the Formula for the Given Equation

Solve for the given variable:

$\boldsymbol{C = 2\pi r}$; **for** $\boldsymbol{r}$

Solution

Step 1: $\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$ (*Division property of equality)*

Step 2: $\frac{C}{2\pi} = r$ *(Simplify)*

Example 2 Solve the Formula for the Given Equation

Solve $P = 2l + 2w$; for w

Solution

Step 1: $P = 2l + 2w$ (*Subtraction property of equality)*

$-2l \quad -2l$

$P - 2l = 2w$

Step 2: $\frac{P-2l}{2} = \frac{2w}{2}$ (*Division property of equality)*

$\frac{P-2l}{2} = w$

Notice that the 2's do not cancel in this problem!

Algebra II Essentials

When dividing both sides of an equation by a variable, you must make sure that the denominator of the fraction you create is not zero.

Step 1: Solve for the variable in question.
Step 2: Look at the denominator in your answer.

- If there is a variable in your denominator, set the denominator equal to zero to find any restrictions.
- If there is no variable in your denominator, the problem is complete.

Example 3 Solve the Formula for the Given Equation

Solve $\boldsymbol{A = \frac{1}{2}h(b_1 + b_2)}$; for $\boldsymbol{h}$

Solution

Step 1: $A = \frac{1}{2}h(b_1 + b_2)$ *(Multiply both sides by 2)*

$2A = \mathbf{2}(\frac{1}{2}h(b_1 + b_2))$

$2A = h(b_1 + b_2)$ *(Simplify)*

Step 2: $\frac{2A}{(b_1+b_2)} = \frac{h(b_1+b_2)}{(b_1+b_2)}$ (*Divide both sides by* $(b_1 + b_2)$*)*

$\frac{2A}{(b_1+b_2)} = h$ *(Simplify)*

Step 3: Look at your denominator. Since there are variables in the denominator, we have to think about restrictions.

$b_1 + b_2 \neq 0$

This is the formula for the area of a trapezoid. If $b_1 + b_2 = 0$, there would be no trapezoid!

Example 4 Solve the Formula for the Given Equation

Solve $\boldsymbol{ax + bx = c}$ for $\boldsymbol{x}$

Solution

Step 1: $ax + bx = c$ (*Use the distributive property to factor out the x)*

$x(a + b) = c$

Step 2: $\frac{x(a+b)}{(a+b)} = \frac{c}{(a+b)}$ (*Divide both sides by* $(a + b)$*)*

$\mathrm{x} = \frac{c}{a+b}$ *(Simplify)*

Step 3: Look at your denominator. Since there are variables in the denominator, we have to think about restrictions.

$a + b \neq 0$

Build Your Skills: Try Isolating a Variable

Solve for the given variable. Don't forget to check for any restrictions.

1. $A = \frac{1}{2}bh$; for h **2.** $I = Prt$; for P

3. $E = mc^2$; for m. **4.** $A = \frac{1}{2}aP$; for P

5. $A = \frac{1}{2}h(b_1 + b_2)$; for b_2

Did you know?

Coaches and scouts, such as a baseball manager, might want to use the formula for earned run average to figure out how many innings a player pitched. They would solve for the variable so that the equation could be used many times.

Career Preparation: Practice

Solve for the given variable. Remember to check for any restrictions.

1. $V = \frac{1}{3}Bh$ for B

2. $C = \frac{5}{9}(F - 32)$ for F

3. $S = 2\pi rh$; for h

4. $y = ax^2 + bx + c$; for b

5. $S = \frac{\pi r^2 a}{360}$; for a

6. $ax + bx = 12$ for x

7. The formula to calculate the average of 5 test scores is $A = \frac{a+b+c+d+e}{5}$. Solve for e, the 5th test score.

8. The formula to calculate the volume of a cylinder is $V = \pi r^2 h$. Solve the equation for *h*, the height of the cylinder.

9. The formula for finding the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$. Solve the equation for *h*, the height of the cylinder.

10. **Error Analysis** Bianca is trying to solve the equation $y = 3x + 2$ for x. Here is what she has done: $y = 3x + 2$

$$\frac{y}{3} = x + 2$$
$$\frac{y}{3} - 2 = x$$

What did she do wrong?

11. **Error Analysis** Cai is trying to solve the equation $cx - b = ax + d$ for x. Here is what he has done: $cx - b = ax + d$

$$cx - ax = d + b$$
$$x(c + a) = d + b$$
$$x = \frac{d+b}{c+a}$$

What did he do wrong?

12. **Writing** The equation for finding the gas mileage of a car is $M = \frac{d}{g}$ where M is the gas mileage, d = distance travelled in miles and g is gallons of gas used. Explain the steps for solving the equation for g, gallons of gas used.

Use It On the Job

13. Emmanuel is driving his car that is equipped with an E-Z Pass from the toll plaza in Carlisle, PA (mileage marker 226 on the PA Turnpike) to the one in Valley Forge (marker 326) in 1 hour and 15 minutes. What was the average speed Emmanuel was driving? Solve the formula $d = rt$ for r (rate) and then plug in Emmanuel's numbers to calculate his average speed. Then, compare your answer to the answer we found in the lesson introduction.

14. Enrique is the coach of a local baseball team. He knows that the formula to find earned run average is $A = \frac{9R}{I}$. A scout gave him a list of pitchers with their earned run averages as well as the number of innings each played. Help the coach calculate the number of earned runs each pitcher has by solving the equation for R.

Career Preparation: Check

1. Solve $y = 3x + 10$ for x

A. $x = \frac{1}{3}y + 10$

B. $x = \frac{1}{3}(y - 10)$

C. $x = \frac{1}{3}y - 10$

D. $3x = y - 10$

2. Solve $A = \frac{1}{2}bh$ for b

A. $b = \frac{2A}{h}$

B. $b = \frac{1}{2}Ah$

C. $b = 2Ah$

D. $b = \frac{A}{2h}$

3. Solve $P = 2l + 25$ for l

A. $l = \frac{P}{2} - 25$

B. $l = P - \frac{25}{2}$

C. $l = \frac{P-25}{2}$

D. $2l = P + 25$

4. Solve $F = ma$ for m

A. $m = Fa$

B. $m = \frac{a}{F}$

C. $m = F + a$

D. $m = \frac{F}{a}$

5. What are the restrictions if $x = \frac{5}{y}$?

A. $y \neq 0$

B. $x \neq 0$

C. $y \neq 5$

D. There are none

6. What are the restrictions if $x = \frac{4b}{7a}$?

A. $b \neq 0$

B. $a \neq 7$

C. $a \neq 0$

D. $b \neq 4$

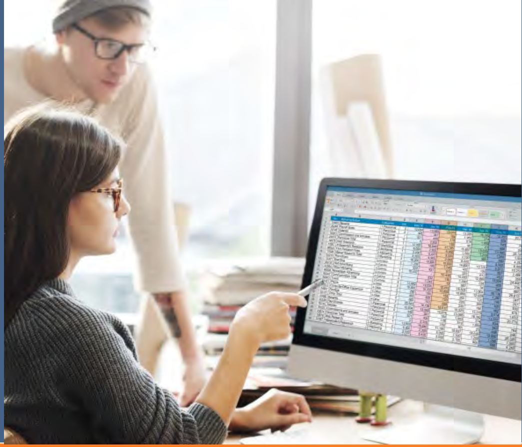
Use It On the Job

7. Diamond is a parking attendant. She knows that the parking ramp she works at uses the formula $C = 0.5t + 2$ where t is measured in hours to calculate how much it costs to park in her ramp. (That means that it costs $2 plus $0.50 per hour). She knows how much Andre paid to park today. Solve the equation for t to help Diamond figure out how long Andre parked.

8. Ahote is an architect that is trying to make a square corner. He knows that the Pythagorean Theorem is $a^2 + b^2 = c^2$ and that if you know the lengths of the legs of a right triangle (*a* and *b*), you can calculate the length of the hypotenuse (*c*). The problem is that Ahote knows the length of one of the legs and the hypotenuse. Help Ahote find the missing leg by solving the equation for b. Hint: solve for b^2 and then take the square root of both sides of the equation.

LESSON 1.6

Apply Solving Rational Equations with More than One Solution



CAREER SPOTLIGHT: Budget Analysts

Occupation Description

Budget analysts evaluate budget proposals, analyze data to determine the costs and benefits of various programs, and recommend funding levels. They oversee spending to ensure that organizations comply with the budget and to determine whether certain programs need changes in funding.

In addition to providing technical analysis, budget analysts must communicate their recommendations effectively within the organization.

Budget analysts working in government may attend committee hearings to explain their recommendations. Occasionally, budget analysts evaluate how well a program is doing, assess policy, and draft budget-related legislation.

Education

A bachelor's degree in fields such as business, finance, or public administration is typically required to become a budget analyst. Some employers prefer to hire applicants who have a master's degree. Sometimes, budget- or finance-related work experience may be substituted for formal education.

Potential Employers

The largest employers of budget analysts are as follows:

Federal government	22%
Educational services	13%
Professional, scientific, and technical services	11%
State government, excluding education and hospitals	11%
Local government, excluding education and hospitals	11%

Watch a video about budget analysts: https://cdn.careeronestop.org/OccVids/OccupationVideos/13-2031.00.mp4

Career Cluster
Finance

Career Pathway
Business Finance

Career Outlook

- Salary Projections:
  Low-End Salary, $51,220
  Median Salary, $78,970
  High-End Salary, $121,360
- Jobs in 2019: 55,400
- Job Projections for 2029: 57,300 (increase of 3%)

Algebra II Concepts

- Demonstrate how budget analysts might apply solving rational equations.

Is this a good career for me?

Budget Analysts typically do the following:

- Prepare financial documents, reports, or budgets.
- Advise others on financial matters.
- Analyze budgetary or accounting data.
- Verify accuracy of financial information.
- Gather financial records.
- Establish organizational guidelines or policies.
- Analyze business or financial data.

Lesson Objective

In this lesson, you will demonstrate how budget analysts might apply solving rational equations with more than one solution.

Formulas Related to Budgets
The following basic formulas may be used by budget analysts. Each formula can be used in the given form or rewritten in a form appropriate for a specific problem.

Earnings for Employee with Hourly Pay Rate
earnings (dollars) = pay rate (dollars per hour) × time worked (hours)

Profit
profit = revenue − expenses

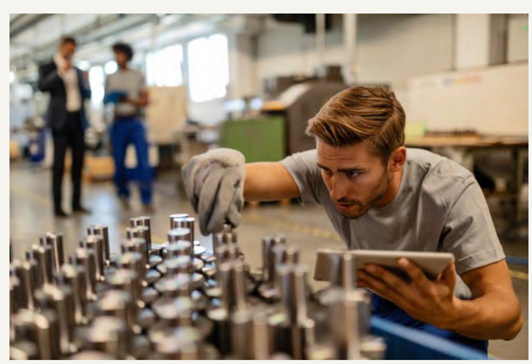
Profit Margin
$$\text{profit margin} = \frac{\text{revenue} - \text{expenses}}{\text{revenue}}$$

Cost Percentages
$$\text{labor cost percent} = \frac{\text{labor cost}}{\text{revenue}} \times 100$$
$$\text{overhead cost percent} = \frac{\text{overhead cost}}{\text{revenue}} \times 100$$

Projected Revenue
projected revenue = previous revenue × (1 + projected growth rate)

1 Step Into the Career: Apply Solving Rational Equations

A steel manufacturing facility would like to hire an experienced technician and a novice technician to work with the experienced technician. A budget analyst must recommend hourly pay rates for the technicians.



To stay within the budget, the experienced technician can earn \$600 per week and the novice technician can earn \$480 per week. The novice technician will be paid \$3 less per hour than the experienced technician.

If the two technicians work a combined total of 80 hours per week, then what is the hourly pay rate of each technician?

Devise a Plan

Step 1: Organize the given information.

Step 2: Write an equation that relates the given information.

Step 3: Solve the equation.

Step 4: Analyze the results in the context of the problem.

Walk Through the Solution

Step 1: Organize the given information. Let r represent the pay rate (in dollars per hour) for the experienced technician.

	Experienced technician	Novice technician
Weekly earnings (dollars)	600	480
Pay rate (dollars per hour)	r	$r - 3$

The two technicians work a combined total of 80 hours per week.

The novice technician earns $3 less per hour than the experienced technician.

Step 2: Write an equation.

Start by relating the time worked by each technician in a verbal formula.

Time worked by experienced technician	+	Time worked by novice technician	=	Total time worked by both technicians

The earnings formula can be rewritten to equal time worked by dividing each side by the pay rate.

$$\text{earnings} = \text{pay rate} \times \text{time worked} \rightarrow \frac{\text{earnings}}{\text{pay rate}} = \text{time worked}$$

Rewrite the original verbal equation by substituting earnings divided by pay rate for time.

Experienced technician		Novice technician		
$\dfrac{\text{earnings (dollars)}}{\text{pay rate (dollars per hour)}}$	+	$\dfrac{\text{earnings (dollars)}}{\text{pay rate (dollars per hour)}}$	=	Total time worked by both technicians

Experienced technician		**Novice technician**		
$\dfrac{600}{r}$	+	$\dfrac{480}{r-3}$	=	80

Use the information in the table in Step 1.

Step 3: Solve the equation $\frac{600}{r} + \frac{480}{r-3} = 80$

$\frac{600}{r} + \frac{480}{r-3} = 80$	Write equation.
$r(r-3)\left(\frac{600}{r} + \frac{480}{r-3}\right) = r(r-3) \bullet 80$	Multiply each side by the LCD, $r(r-3)$.
$\frac{r(r-3)(600)}{r} + \frac{r(r-3)(480)}{r-3} = r(r-3) \bullet 80$	Use Distributive Property.
$(r-3)(600) + r(480) = r(r-3) \bullet 80$	Simplify.
$600r - 1800 + 480r = 80r^2 - 240r$	Simplify.
$80r^2 - 1320r + 1800 = 0$	Write in standard form.
$40(2r^2 - 33r + 45) = 0$	Divide out common factor.
$40(2r-3)(r-15) = 0$	Factor.
$r = 1.5 \quad \text{or} \quad r = 15$	Use the Zero Product Property.

The solutions are 1.5 and 15.

Step 4: Analyze the solutions.

First, check for extraneous solutions by checking the solutions in the original equation.

Check r = 1.5.	**Check r = 15.**
$\frac{600}{r} + \frac{480}{r-3} = 80$	$\frac{600}{r} + \frac{480}{r-3} = 80$
$\frac{600}{1.5} + \frac{480}{1.5-3} = 80$	$\frac{600}{15} + \frac{480}{15-3} = 80$
$400 + (-320) = 80$	$40 + 40 = 80$
$80 = 80$ ✓	$80 = 80$ ✓

Both values produce true statements, so neither of the solutions are extraneous.

Next, check the solutions in the context of the problem.

***r* = 1.5**

If $r = 1.5$, then $r - 3 = 1.5 - 3 = -1.5$. So, the pay rates are as follows:

Experienced technician's pay rate = \$1.50 per hour
Novice technician's pay rate = –\$1.50 per hour

This solution does not make sense in the context of the problem since a person cannot have a negative pay rate.

***r* = 15**

If $r = 15$, then $r - 3 = 15 - 3 = 12$. So, the pay rates are as follows:

Experienced technician's pay rate = \$15 per hour
Novice technician's pay rate = \$12 per hour

This solution make sense in the context of the problem.

The pay rate for the experienced technician should be \$15 per hour, and the pay rate for the novice technician should be \$12 per hour.

On the Job: Apply Solving Rational Equations

1. A company that makes and sells dog collars sold 2500 dog collars for \$10 each this month. The company wants to increase the price of the dog collars but keep the price less than \$20. They estimate that they will lose 100 sales for each \$1 increase in the price per collar.

The company does not want the price change to affect their advertising cost budget, so the budget analyst must recommend a new price for this situation.

In the budget, 15% of sales revenue is used for advertising costs, and the company plans to spend \$4290 on advertising next month.

a. The revenue from the dog collar sales is given by $(10 + x)(2500 - 100x)$ where x represents each \$1 increase in the price of a collar. Use this expression for sales revenue along with the verbal equation below to write an equation.

$$\text{Advertising cost percent (as a decimal)} = \frac{\text{Advertising cost}}{\text{Sales revenue}}$$

b. Solve the equation from part (a) and analyze the solutions. What dog collar price should the budget analyst recommend? Explain your reasoning.

Career Spotlight: Practice

2. A food packaging company would like to hire a supervisor and a packager to work together on a new production line. A budget analyst must recommend hourly pay rates for the workers.

To stay within the budget, the supervisor can earn \$690 per week and the packager can earn \$520 per week. The supervisor will be paid \$10 more per hour than the packager. If the supervisor and the packager work a combined total of 70 hours per week, then what are the hourly pay rates for the supervisor and the packager?

Devise a Plan

Step 1: Organize the given information.

Step 2: ____?____

Step 3: ____?____

Step 4: ____?____

3. A company that makes customized T-shirts sold 1600 T-shirts for \$24 each this month. The company wants to increase their profit margin by increasing their sales and decreasing the T-shirt price but keep the price above \$18. They estimate that they will gain 150 sales for each \$1 decrease in the price per T-shirt.

The company wants to increase the profit margin for T-shirt sales to 40% next month. They expect to have \$26,400 in expenses next month. The budget analyst must recommend a T-shirt price for this situation.

The T-shirt company wants to have a profit margin of 40% next month.

a. The revenue from the T-shirt sales is given by $(24 - x)(1600 + 150x)$ where x represents each \$1 decrease in the price of a T-shirt. Use this expression for sales revenue along with the verbal equation for the profit margin to write an equation.

$$\text{Profit margin (as a decimal)} = \frac{\text{Sales revenue} - \text{Expenses}}{\text{Sales revenue}}$$

b. Solve the equation from part (a) and analyze the solutions. What T-shirt price should the budget analyst recommend? Explain your reasoning.

4. Consider the situation in Exercise 3. Suppose the T-shirt company does not have a profit margin goal. However, they do not want the new T-shirt price to affect their overhead cost budget.

In the budget, 10% of sales revenue is used for overhead costs, and the company plans to spend $4180 on overhead costs next month. What T-shirt price greater than $18 should the budget analyst recommend? Explain your reasoning.

Career Spotlight: Check

5. An earth materials company has a new machine for crushing stone. The new machine and the old machine will operate together for a total of 120 hours a week. In the operating budget, $1260 has been allocated for the weekly operating costs of the new machine, and $1250 has been allocated for the operating costs of the old machine. This information is summarized in the table, where t represents the weekly operation hours of the new machine.

	New machine	**Old machine**
Budgeted weekly operating cost (dollars)	1260	1250
Weekly operation (hours)	t	$120 - t$

Which equation can the budget analyst use to make a recommendation for the number of weekly operating hours for each machine?

A. $1260t + 1250(120 - t) = 120$

B. $\left(\frac{1260}{t}\right)\left(\frac{1250}{120 - t}\right) = \frac{2510}{120}$

C. $\frac{1260}{t} = \frac{1250}{120 - t}$

D. $\frac{1260}{t} + \frac{1250}{120 - t} = \frac{2510}{120}$

6. A company that makes and sells automobile parts wants to increase their profit margin to 30% this month. Last month, they had a sales revenue of $80,500 and had $60,375 in expenses. They expect that they will have the same expenses this month. If x represents the percent increase in sales revenue needed for a profit margin of 30% this month, then which equation can be used to find the value x?

A. $0.3 = \frac{80{,}500(1+x)-60{,}375(1+x)}{80{,}500(1+x)}$

B. $0.3 = \frac{80{,}500(1+x)-60{,}375}{80{,}500(1+x)}$

C. $0.7 = \frac{80{,}500-60{,}375}{80{,}500(1+x)}$

D. $0.3 = \frac{80{,}500x}{80{,}500x-60{,}375}$

7. A company that makes and sells smartphone cases sold 4500 cases for $15 each this month. The company wants to increase their smartphone case price, but keep the price less than $20. They estimate that they will lose 200 sales for each $1 increase in the price per case.

In the budget, 8% of sales revenue is budgeted for rent, and the company plans to spend $5576 on rent next month. The company does not want the price change to affect their budget for rent.

The revenue from the smartphone case sales is given by

a. $(15 - x)(4500 - 200)$
b. $(15 + x)(4500 - 200x)$
c. $(15x)(4500 - 200x)$

where x represents each $1 increase in the price of a case.

An equation that can be used to determine the number of $1 increases in price that will not affect the rent budget is

a. $0.08 = \frac{5576}{(15+x)(4500-200x)}$

b. $0.8 = \frac{(15+x)(4500-200x)}{5576}$

c. $0.8 = \frac{5576}{(15-x)(4500-200)}$

The solutions of the equation are

a. $x = 1$ and 11
b. $x = -2$ and 2
c. $x = 2$ and 5.5

So, the budget analyst should recommend that the company change the price of the smartphone cases to

a. \$14 per case
b. \$16 per case
c. \$17 per case

8. A package delivery service would like to hire another driver and a driver's assistant. A budget analyst must recommend hourly pay rates for the workers.

To stay within the budget, the driver can earn \$960 per week and the assistant can earn \$560 per week. The assistant will be paid \$8 less per hour than the driver. If the driver and the assistant work a combined total of 75 hours per week, then what is the hourly pay rate for the assisstant?

A. \$12 per hour

B. \$16 per hour

C. \$20 per hour

D. \$24 per hour

LESSON 1.6 Teacher Edition

Apply Solving Rational Equations with More than One Solution

CAREER SPOTLIGHT: Budget Analysts

Encourage your students to learn more about this occupation and many more in the Pathway2Careers Career Library.

CAREER SPOTLIGHT: Budget Analysts

Budget analysts prepare budget reports, monitor organizational spending, and advise organizations about the details of their finances. They may work for government organizations, private companies, or universities. They analyze data to determine the costs and benefits of various programs, and they recommend funding levels based on their findings. Employers generally require that budget analysts have at least a bachelor's degree in fields such as business, finance, or public administration. Sometimes, budget- or finance-related work experience may be substituted for formal education.

- Discuss being a budget analyst by reading the Career Spotlight together.
- Find college programs that offer degrees in being a budget analyst and learn their prerequisite skills.
- Contact a finance company and discuss the field of business finance and being a budget analyst. Share your findings with your class.

Video: Budget Analyst

Have students watch this video, which describes the work of a budget analyst.

https://cdn.careeronestop.org/OccVids/OccupationVideos/13-2031.00.mp4

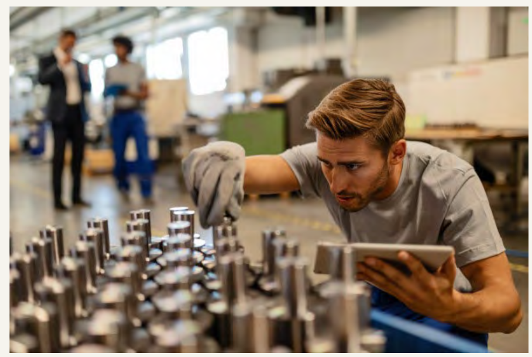
Lesson Objective

Demonstrate how budget analysts might apply solving rational equations with more than one solution.

Teaching Support

1 Step Into the Career: Apply Solving Rational Equations

A steel manufacturing facility would like to hire an experienced technician and a novice technician to work with the experienced technician. A budget analyst must recommend hourly pay rates for the technicians.



To stay within the budget, the experienced technician can earn \$600 per week and the novice technician can earn \$480 per week. The novice technician will be paid \$3 less per hour than the experienced technician.

If the two technicians work a combined total of 80 hours per week, then what is the hourly pay rate of each technician?

Guiding Questions

- In Step 1, suppose r represents the pay rate (in dollars per hour) for the novice technician. In this situation, what algebraic expression represents the pay rate (in dollars per hour) for the experienced technician?
- In Step 2, why is the earnings formula rewritten?
- In Step 4, why is the solution r = 1.5 not considered when answering the question?

ENRICHMENT In the example, the quadratic equation is solved by factoring. Remind students that the quadratic equation can also be solved using the Quadratic Formula or by completing the square. Have students solve the quadratic equation using another method.

DIFFERENTATION: ADDITIONAL SUPPORT Students may benefit from a discussion about how to determine the lowest common denominator of the rational equation in Step 3.

On the Job: Apply Solving Rational Equations

Answers

1a. Sample answer: $0.15 = \frac{4290}{(10+x)(2500-10x)}$

1b. \$13; The solutions of the equation are $x = 3$ and $x = 12$. When $x = 3$, the new price is \$13. When $x = 12$, the new price is \$22. The problem states that the price must be less than \$20, so \$13 is the answer.

Use the questions to check students' understanding:

- Why does $(10 + x)(2500 - 10x)$ represent the sales revenue?
- What is your first step in solving the equation $0.15 = \frac{4290}{(10+x)(2500-10x)}$?

Career Spotlight: Practice

Solution Steps for Exercises 2–4

These steps will help guide students in solving these practice exercises.

Exercise 2

Answer

2. Supervisor is paid \$23 per hour and packager is paid \$13 per hour.

Sample plan:

Devise a Plan

Step 1: Organize the given information.

Step 2: Write an equation that relates the given information.

Step 3: Solve the equation.

Step 4: Analyze the results in the context of the problem.

Solution Steps

- Organize the given information in a table like the one shown.

	Supervisor	Packager
Weekly earnings (dollars)	690	520
Pay rate (dollars per hour)	$r + 10$	r

- Use the given information to write an equation such as $\frac{690}{r+10} + \frac{520}{r} = 70$.
- Solve the equation. The solutions are $r = 13$ and $r \approx -5.7$.
- Check the results for extraneous solutions, and then determine if the results make sense in the context of the problem. The pay rate cannot be negative, so $r \approx -5.7$ cannot be used to answer the question. So, $r = 13$ and $r + 10 = 23$.

Exercise 3

Answer

3a. Sample answer: $0.4 = \frac{(24-x)(1600+150x)-26{,}400}{(24-x)(1600+150x)}$

3b. \$20; The solutions of the equation are $x = 4$ and $x \approx 9.3$. When $x = 4$, the new price is \$20. When $x \approx 9.3$, the new price is about \$14.70. The problem states that the price must be greater than \$18, so \$20 is the answer.

Solution Steps

- To write the equation in part (a), substitute 0.4 for the profit margin, $(24 - x)(1600 + 150x)$ for the sales revenue, and 26,400 for the expenses.
- Cross multiply to solve the equation. The solutions are $x = 4$ and $x \approx 9.3$.
- Analyze the results. Only $x = 4$ makes sense in the context of the problem.

Exercise 4

Answer

4. $22; Using the equation 0.1 = $\frac{4180}{(24-x)(1600+150x)}$, the solutions are *x* = 2 and *x* ≈ 11.3. When *x* = 2, the new price is $22. When *x* ≈ 11.3, the new price is about $12.70. The problem states that the price must be greater than $18, so $22 is the answer.

Solution Steps

- To write an equation, use 0.1 for the overhead percent and 4180 for the overhead costs: 0.1 = $\frac{4180}{(24-x)(1600+150x)}$.
- Cross multiply to solve the equation. The solutions are *x* = 2 and *x* ≈ 11.3.
- Analyze the results. Only *x* = 2 makes sense in the context of the problem.

Career Spotlight: Check

Tips for Completing Exercises 5–8

These tips will help students in solving these exercises and similar assessment items.

Exercise 5

Answer

5. D

Tip Write a general verbal equation that relates the given information. For example, use the following to write an equation:

Operation cost rate for new machine (dollars per hour)	+	Operation cost rate for old machine (dollars per hour)	=	Total operation cost rate (dollars per minute

Exercise 6

Answer

6. B

Tip Eliminate choices that are obviously incorrect. For example, profit margin is the following ratio: $\frac{\text{revenue} - \text{expenses}}{\text{revenue}}$. This format is not used in choice D, so that choice can be eliminated.

Exercise 7

Answer

7. b; a; c; c

Tip Encourage students to keep track of their answers in an organized way. Given the number of choices in this problem, students may write their answer choices down in the wrong order.

Exercise 8

Answer

8. B

Tip Remind students to check the solutions of their equation in the context of the problem.